ABSTRACT

The purposes of this research are to apply a mathematical model called the convection-diffusion equation

\[-D \frac{d^2 C}{dx^2} + u \frac{dC}{dx} + RC - Q = 0,\]

and a finite difference method

\[y'' = p(x)y' + q(x)y + r(x),\]

to the water pollution approximation problem in a uniform channel by modifying some parameters of the above methods and to formulate a constrained optimization model to keep the pollution levels and the associated budgets within acceptable ranges. In general, the pollution levels are measured via field data collection which is often complicated and erroneous at some data sources.

The processes of this research begin by using a finite difference method to find a numerical solution of a second-order linear ordinary differential equation with Dirichlet and Neumann boundary conditions. Then a new finite difference method is proposed to solve the convection-diffusion equation for approximating the water pollution concentration levels in the uniform channel. The obtained solutions are in the form of linear equations. And then, these solutions are used to formulate an optimization model, of which the objective function is to minimize the costs of water treatment and the constraints include the legal regulations and the planned budgets of the factory.

The results of this research show that the modified convection-diffusion equation and the finite difference method are suitable to water pollution level approximation in a uniform channel
and the numerical solution
\[ (-2)w_{N-1} + (2 + h^2 q(x_N) w_N) = -h^2 r(x_N) + (2 - hp(x_N)) h \beta, \]
used to formulate the optimization model, assists the factory in controlling the expenses of water treatment while the pollution levels are in the legal regulations. The proposed techniques and methods can also be applied to the water pollution approximation problem in a uniform channel of other factories.

**Keywords**: Numerical computation, Water quality measurement, Uniform channel, Convection-diffusion equation, Optimization